

BOURGAIN ALGEBRAS ON THE MAXIMAL IDEAL SPACE OF H^∞

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ABSTRACT. Let B be a Douglas algebra. For another Douglas algebra D , by considering the integral representation, there exists the corresponding closed subspace \hat{D} of $C(M(B))$ the space of continuous functions on maximal ideal space of B . Let $[\hat{D}]_{M(B)}$ be the closed subalgebra of $C(M(B))$ generated by \hat{D} . In this paper we describe the algebra $[\hat{D}]_{M(B)}$ and determine the Bourgain algebra of $[\hat{D}]_{M(B)}$ relative to $C(M(B))$.

1. Introduction. The concept of Bourgain algebras was introduced by Cima and Timoney (see [2] and [6]). Let \mathcal{A} be a Banach algebra with identity, and let \mathcal{B} be a closed subalgebra of \mathcal{A} . The *Bourgain algebra* \mathcal{B}_b relative to \mathcal{A} is the space of f in \mathcal{A} such that $\|f f_n + \mathcal{B}\| \rightarrow 0$ ($n \rightarrow \infty$) for every sequence $\{f_n\}_n$ in \mathcal{B} converging weakly to zero. Cima and Timoney proved that \mathcal{B}_b is a closed subalgebra of \mathcal{A} containing \mathcal{B} . We shall write \mathcal{B}_{bb} for $(\mathcal{B}_b)_b$. For other recent papers on Bourgain algebras, the reader is referred to [4, 5, 9, 10, 13, 17, 20, 22, 23].

Let H^∞ be the space of boundary functions of bounded analytic functions on the open unit disk Δ . With the essential supremum norm, H^∞ is a subalgebra of L^∞ on the unit circle T . A closed subalgebra B of L^∞ containing H^∞ is called a Douglas algebra. Let C denote the space of continuous functions on T . As Sarason showed, the algebra $H^\infty + C$ is a Douglas algebra (see [21] for a discussion of this algebra). In [4], Cima, Janson and Yale proved that $(H^\infty)_b = (H^\infty)_{bb} = H^\infty + C$ relative to L^∞ . In [10], the authors and Mortini studied Bourgain algebras of Douglas algebras B and showed that $B_b = B_{bb}$ relative to L^∞ . In [17] the second author determined the Bourgain algebra of the disk algebra A and proved that $A_b = A_{bb}$ relative to L^∞ . These are all studies of Bourgain algebras relative to L^∞ on T .

In what follows we denote the set of nonzero multiplicative linear functionals of a Douglas algebra B by $M(B)$. With the weak*-topology,

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