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ASPLUND SPACES AND DECOMPOSABLE NONSEPARABLE BANACH SPACES

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ABSTRACT. We show that an Asplund space of density character $\aleph_1$ is weakly compactly generated if and only if it has a projectional resolution of identity for each equivalent norm. We show that every nonseparable Asplund space has a nonseparable subspace which has an equivalent strictly convex norm. We give an example of a non-Asplund space such that every bounded weakly closed subset is an intersection of finite union of balls. We show the existence of an Eberlein compact K such that $(\mathcal{C}(K), ||.||_\infty)$ has no λ-norming Markushevich basis if $\lambda < 2$.

0. Introduction. In this note we investigate some properties of the nonseparable Banach spaces which admit a "decomposition" into separable subspaces. We show, for instance, that there exists a weakly compactly generated (wcg) Banach space X with no λ-norming Markushevich basis for $\lambda < 2$, and in fact that there exists an Eberlein compact K such that $(\mathcal{C}(K), ||.||_\infty)$ has this property. This improves some results from [**18**]. We also answer a question from [**8**].

Let us recall some notation. Let X be a Banach space of density character $\operatorname{dens}(X) = \mu$. A "decomposition" of X is a well-ordered collection $\{P_\alpha; \omega_0 \leq \alpha \leq \mu\}$ of projections such that $P_\alpha P_\beta = P_\beta P_\alpha = P_\alpha$ if $\alpha \leq \beta$, $P_\mu = \operatorname{Id}_X$, $P_\beta(x) \in \overline{\{P_{\alpha+1}(x); \alpha < \beta\}}$ for all $x \in X$ and β, and $\operatorname{dens}(P_\alpha(X)) \leq |\alpha|$ for all α. The decomposition $\{P_\alpha; \omega_0 \leq \alpha \leq \mu\}$ is called a *projectional resolution of identity* (PRI) if $||P_\alpha|| \leq 1$ for all α. It is called a *separable decomposition* if $(P_{\alpha+1} - P_\alpha)(X)$ is separable for all $\alpha < \mu$.

Jayne-Rogers selectors were shown to exist in [**13**] (see [**2**, Chapter I.4]). They are multivalued maps from Asplund spaces X to the set $(X^*)^{\mathbf{N}}$ of countable subsets of X^*. We denote them by Δ. A subset $Y \subset X^*$ is called (λ)-norming if there exists $\lambda < \infty$ such that

$$||x|| \leq \lambda \sup\{|f(x)|; f \in Y, ||f|| \leq 1\}$$

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