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EXPONENTIAL DICHOTOMIES IN LINEAR SYSTEMS WITH A SMALL PARAMETER

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1. Introduction. Using Melnikov's technique in bifurcation theory, we investigate the theory of exponential dichotomy in linear systems depending on a real parameter. We generalize a well-known result on the exponential dichotomy in K.J. Palmer [**8**]. It is well known that the theory of exponential dichotomy plays very important roles in studying nonautonomous dynamical systems and has received much attention. For example, on the stability theory, we refer to Coppel [**2**]; on the existence of almost periodic solutions, to Fink [**3**]; on the theory of topological equivalence, to Palmer [**7**]; on the bifurcation theory and chaos, to Meyer and Sell [**5, 6**], Palmer [**8, 9**] and Battelli and Palmer [**1**]. About the theory of exponential dichotomy, we refer to Sacker and Sell [**10**] and Coppel [**2**].

We consider a linear differential equation

$$\dot{x} = A(t)x \tag{1}$$

where $x \in R^n$ and $A(t)$ is an $n \times n$ continuous bounded matrix defined on R. We say that the linear differential equation (1) admits an exponential dichotomy with constants K and α on an interval J if there exist a projection P and the fundamental matrix, denoted by $X(t)$, of equation (1) satisfying

$$\begin{aligned} |X(t)PX^{-1}(s)| &\le Ke^{-\alpha(t-s)}, \qquad t \ge s \\ |X(t)(I-P)X^{-1}(s)| &\le Ke^{-\alpha(s-t)}, \qquad s \ge t \end{aligned}$$

for $t, s \in J$. In particular, when $A(t) = A$ is a constant matrix, equation (1) possesses an exponential dichotomy on R if and only if the real parts of the eigenvalues of the matrix A are different from zero. We are only interested in exponential dichotomies on $J = R$, R^+ and R^-.

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