ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 25, Number 4, Fall 1995

ON THE SOLUTIONS OF FOURTH ORDER DIFFERENCE EQUATIONS

J. POPENDA AND E. SCHMEIDEL

1. Introduction. In this note we will study fourth order difference equations of the form

$$\Delta^4 y_n = f(n, y_{n+2}), \qquad n \in \mathbf{N}. \tag{E}$$

We denote by $\mathbf{N}$ the set of positive integers, by $\mathbf{R}$ the set of real numbers. For a function $x : \mathbf{N} \to \mathbf{R}$, the forward difference operators are defined as follows: $\Delta x_n = x_{n+1} - x_n$, $n \in \mathbf{N}$ and $\Delta^k x_n = \Delta(\Delta^{k-1} x_n)$ for $k > 1$.

By a solution of (E) we mean any sequence $y = \{y_n\}_{n=1}^{\infty}$ which satisfies (E) for all $n \in \mathbf{N}$. We call the solution y the zero (or trivial) solution if it is identically zero or if there exists $\nu \in \mathbf{N}$ such that $y_n = 0$ for all $n \geq \nu$.

A nonzero solution is oscillatory if, for every $m \in \mathbf{N}$ there exists $n \geq m$ such that $y_n y_{n+1} \leq 0$. Therefore, a nonoscillatory solution is such a sequence, which is eventually positive or eventually negative. We suppose that the function $f : \mathbf{N} \times \mathbf{R} \to \mathbf{R}$ satisfies condition $(*)$ if

$$x f(n, x) < 0 \quad \text{for all } n \in \mathbf{N},\ x \in \mathbf{R} \backslash \{0\}. \tag{$*$}$$

In his paper [**2**], W. Taylor considered two types of solutions of the fourth order linear difference equations (see also [**1**]). Relations between these types of solutions for the equation (E), and their oscillatory behavior are the main purposes of this note. Some of our theorems generalize results in the work of Taylor. This refers to Theorems 1, 2, 3 and 4 proved below and Theorems 1.3, 2.3, 2.4 and 2.5 of [**2**], respectively.

Following Taylor, we define operator F as

$$F(x_n) = x_{n+1} \Delta^3 x_n - \Delta x_n \Delta^2 x_n, \qquad n \in \mathbf{N}.$$

Received by the editors on July 17, 1993, and in revised form on April 11, 1994.

Copyright ©1995 Rocky Mountain Mathematics Consortium