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RESONANT SINGULAR BOUNDARY VALUE PROBLEMS

DONAL O'REGAN

ABSTRACT. Existence theory is developed for the "resonant" singular problem $(1/(pq))(py')' + \lambda_0 y = f(t, y, py')$ almost everywhere on $[0,1]$ with $\lim_{t\to 0^+} p(t)y'(t) = ay(1) + b\lim_{t\to 1^-} p(t)y'(t) = 0$. Here λ_0 is the first eigenvalue of $(1/(pq))(pu')' + \lambda u = 0$ almost everywhere on $[0,1]$ with $\lim_{t\to 0^+} p(t)u'(t) = au(1) + b\lim_{t\to 1^-} p(t)u'(t) = 0$. We do *not* assume $\int_0^1 ds/p(s) < \infty$ in this paper.

1. Introduction. This paper presents existence results for the second order singular "resonant" boundary value problem

$$(1.1)\qquad \begin{cases} \frac{1}{pq}(py')' + \lambda_0 y = f(t, y, py'), & \text{a.e. on } [0,1] \\ \lim_{t\to 0^+} p(t)y'(t) = 0 \\ ay(1) + b\lim_{t\to 1^-} p(t)y'(t) = 0, & a > 0,\ b \geq 0 \end{cases}$$

where λ_0 is the *first* eigenvalue (described in more detail later) of

$$(1.2)\qquad \begin{cases} Lu = \lambda u, & \text{a.e. on } [0,1] \\ \lim_{t\to 0^+} p(t)u'(t) = 0 \\ au(1) + b\lim_{t\to 1^-} p(t)u'(t) = 0, & a > 0,\ b \geq 0 \end{cases}$$

with $Lu = -(1/(pq))(pu')'$.

Throughout the paper $p \in C[0,1] \cap C^1(0,1)$ together with $p > 0$ on $(0,1)$; also q is measurable with $q > 0$ almost everywhere on $[0,1]$ and $\int_0^1 p(x)q(x)\,dx < \infty$.

Remark. We do *not* assume $\int_0^1 ds/p(s) < \infty$ in this paper.

Also $pqf : [0,1] \times \mathbf{R}^2 \to \mathbf{R}$ is an L^1-Caratheodory function. By this, we mean:

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