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REGULAR SOBOLEV TYPE ORTHOGONAL POLYNOMIALS: THE BESSEL CASE

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ABSTRACT. In this paper, given a regular linear functional u on the linear space $\mathbf{P}$ of polynomials with real coefficients, we consider the bilinear symmetric form $\varphi(p,q) = \langle u, pq\rangle + \lambda p'(c)q'(c)$ where λ and c are real numbers and $p,\ q \in \mathbf{P}$. A necessary and sufficient condition to warrant the existence of a sequence of orthogonal polynomials with respect to φ is given, and different expressions in terms of the orthogonal polynomials associated to u are studied. Also, we consider the relations between these polynomials and the orthogonal polynomials associated to the linear functional $u_1 = (x-c)^2 u$. Finally, we illustrate these ideas with a nontrivial example, the functional associated to the Bessel polynomials.

1. Introduction. In the last years, a nonstandard class of orthogonal polynomials has attracted considerable attention. The so-called *Sobolev type orthogonal polynomials* (see references [**2, 3, 5, 9, 10**]) are associated to inner products like

$$(p,q)_w = \sum_{k=0}^{N} \int_{\mathbf{R}} p^{(k)}(x) q^{(k)}(x)\, d\mu_k(x)$$

where μ_0 is a finite positive Borel measure and μ_k, $k = 1, \ldots, N$ are discrete measures.

In this work, if u is a regular linear functional on the linear space $\mathbf{P}$ of polynomials with real coefficients, we shall consider the bilinear symmetric form

$$\varphi(p,q) = \langle u, pq\rangle + \lambda p'(c)q'(c)$$

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