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BIQUADRATIC RESIDUES AND SELF-ORTHOGONAL 2-SEQUENCINGS

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1. Introduction. Let $2K_n$ denote the complete multigraph on n vertices in which each edge has multiplicity two. If $2K_n$ can be partitioned into Hamiltonian paths such that any two distinct paths have exactly one edge in common, write $2K_n \to P_n$. The object of this paper is to examine a particular class of such partitions introduced in [**2**], to which the reader is referred for wider discussion of this and related graph decomposition problems. In particular, attention is paid to constructions of these partitions that are based on self-orthogonal 2-sequencings of the additive groups of finite fields.

Definition 1. Suppose H is a finite group of order n with identity element e. A 2-*sequencing* (or *terrace*) of H is an ordering $e, c_2, \dots, c_n$ of elements of H (not necessarily distinct) such that

(i) the partial products $e, ec_2, ec_2c_3, \dots, ec_2\cdots e_n = e, d_2, \dots, d_n$ are distinct (and hence all of H),

(ii) if $h \neq h^{-1}$, then $|\{i : 2 \le i \le n \text{ and } (c_i = h \text{ or } c_i = h^{-1})\}| = 2$,

(iii) if $h = h^{-1}$, then $|\{i : 1 \le i \le n \text{ and } c_i = h\}| = 1$.

As all groups considered in this paper are abelian, additive notation is used. An example taken from [**2**] on $\mathbf{Z}_{19}$ is useful as an illustration.

Consider, on $\mathbf{Z}_{19}$, the 2-sequencing S, and associated partial sum sequence P, as follows:

$$\begin{aligned} S:&\quad 0, 2, 5, 6, 4, 1, 7, -3, -2, 9, 6, -8, 1, 5, -3, 4, 9, 7, -8 \\ P:&\quad 0, 2, 7, 13, 17, 18, 6, 3, 1, 10, 16, 8, 9, 14, 11, 15, 5, 12, 4 \end{aligned}$$

The sequence P can be thought of as a Hamiltonian path through the vertices of K_{19}, the complete graph with vertices labelled by elements

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