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IRRATIONAL SUMS

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1. Introduction. In this note we give some sufficient conditions for the irrationality of the sum of the series $\sum_{n=1}^{\infty} 1/H(f(n))$, where $(H(k))_{k\geq 0}$ is a sequence of integers, positive from some point on, satisfying a homogeneous linear recurrence relation with integer coefficients, and f is a strictly increasing function from the set of positive integers to the set of nonnegative integers.

We will refer to such a sequence $(H(k))_{k\geq 0}$ simply as a "recurrent sequence," and the symbol f will always denote a strictly increasing function from the set of positive integers to the set of nonnegative integers.

Let us agree that the symbol $\sum 1/H(f(n))$ denotes the summation of all those terms $1/H(f(n))$ for which $H(f(n)) > 0$.

All of our results are based on the following theorem of C. Badea [**1**].

Theorem A (Badea [**1**]). *If $(a_k)_{k\geq 0}$ is a sequence of positive integers such that $a_{k+1} > a_k^2 - a_k + 1$ for all sufficiently large k, then $\sum 1/a_k$ is irrational.*

A simple example to show that the converges of Badea's Theorem A is false is the series $\sum 1/n! = e$. Another easy example to see that the converse of Badea's result is false is the following. Let $\{c_n\}$, $n \geq 1$, be a nonperiodic sequence of 2s and 5s, and let $a_n = 10^n/c_n$, $n \geq 1$. Then $\sum 1/a_n$ is irrational, and $a_{n+1} \leq a_n^2 - a_n + 1$, $n \geq 3$.

Thus our goal is to find simple conditions on $H(k)$ and $f(n)$ which ensure that $H(f(n+1)) > H(f(n))^2 - H(f(n)) + 1$ for all sufficiently large n.

To avoid complications, *from now on we will always assume that the characteristic polynomial of the recurrent sequence $H(k)$ has a unique*

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