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EQUIMEASURABLE REARRANGEMENTS OF FUNCTIONS AND FOURTH ORDER BOUNDARY VALUE PROBLEMS

PHILIP SAVOYE

ABSTRACT. The buckling properties of a vibrating beam subject to axial compressive and elastic destructive forces are investigated in this paper. In particular, lower bounds for the eigenvalues of the corresponding boundary value problem are obtained and expressed in terms of equimeasurable rearrangements of the associated differential equation's coefficients.

1. Introduction. In this paper, we investigate the buckling properties of a vibrating beam. Our interest in this fourth order boundary value problem stems from earlier work done by Barnes [**2, 3**] in obtaining spectral inequalities for second and fourth order problems.

The beam investigated in this paper has stiffness $p(x)$ and is subject to an axial compressive load λ which causes it to buckle. The beam is supported on an elastic foundation which provides, at each point x, an elastic destructive force $F(x)y$, $F(x) < 0$, which opposes restoration toward the line of no deflection and is directly proportional to the displacement y. From elementary beam theory, the natural modes of buckling of our problem are the eigenfunctions of the differential equation

$$(p(x)y'')''(x) + \lambda y''(x) + F(x)y = 0, \qquad x \in (0, l) \tag{1}$$

subject to elastically constrained boundary conditions. We will assume hinged-hinged boundary conditions

$$y(0) = y(l) = y''(0) = y''(l) = 0. \tag{2}$$

It can be shown [**8, 1**] using a Prüffer transformation that the nth eigenfunction y_n has $n-1$ zeros η_i interlaced as follows

$$0 = \eta_1 < \eta_2 < \cdots < \eta_{n-1} = l. \tag{3}$$

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