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PERTURBATION ANALYSIS OF A SEMILINEAR PARABOLIC PROBLEM WITH NONLINEAR BOUNDARY CONDITIONS

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ABSTRACT. We consider a diffusion model in which a distributed nonlinear absorption mechanism competes with a nonlinear boundary source. By assuming both these nonlinearities to be weak, a formal asymptotic approximation can be constructed to describe the magnitude and stability of the different responses that can occur in different parameter regimes.

1. Introduction. Our objective is the study of the behavior of positive solutions of the nonlinear, parabolic, initial-boundary value problem:

$$u_t = u_{xx} - \lambda u^p, \qquad x \in (0,1),\ t > 0, \tag{1.1a}$$

$$u_x(0,t) = 0, \qquad u_x(1,t) = u^q(1,t), \qquad t > 0, \tag{1.1b}$$

$$u(x,0) = u_0(x), \tag{1.1c}$$

where λ, p, q are constants $\lambda > 0$, $p > 1$, $q > 1$, and $u_0 > 0$.

The problem (1.1) is a generalization of some other nonlinear diffusion problems. One example is the question of global existence of positive solutions to the heat equation $u_t = \Delta u$, $(x,t) \in \Omega \times \mathbf{R}^+$, Ω a bounded domain in $\mathbf{R}^N$, subject to the nonlinear boundary condition $\partial u/\partial \nu = f(u)$ on $\partial\Omega$ (ν being the exterior unit normal to $\partial\Omega$) and with a nonnegative initial condition $u(x,0) = u_0(x)$. The main feature of this problem is the general tendency of positive solutions to blow up in finite time provided that $f = f(u)$ is a superlinear function. This blow-up property was first proved for this problem in [**4**] for $f(u)$ a power u^q, $q > 1$, and certain large enough initial data u_0 (see also [**7**] for early related but complementary global existence results). In [**5**] the blow-up property was proved for all nonnegative initial data u_0 provided that $f(u) = u^q$ and either $q > 1$, $N = 1, 2$, or $1 < q < N/(N-2)$ for $N > 2$

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