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ECOLOGICAL MODELS, PERMANENCE AND SPATIAL HETEROGENEITY

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ABSTRACT. We model the interactions of two theoretical populations which are allowed to move at random throughout a bounded habitat via systems of two weakly coupled reaction-diffusion equations. The reaction terms in these systems involve parameters which are subject to biological interpretation and which are assumed to be spatially dependent. We examine the effect of spatial heterogeneity on the long-term viability of each of the populations, with the aim of quantifying the effect in terms of the biological parameters in the models. To this end, we employ the dynamic concept of permanence of the interacting populations, conditions for which lead directly to the spectral theory for linear elliptic boundary value problems, so that the long-term viability of the populations can be expressed in terms of eigenvalues depending on the biological parameters of the models in directly quantifiable ways. We give a number of examples and demonstrate for the first time via reaction-diffusion equations that spatial heterogeneity can lead to coexistence in situations wherein extinction would result, were the habitat spatially homogeneous.

1. Introduction. In this article we continue the examination of the long-term dynamics of two interacting species which was begun in [**14**]. We assume that the species are free to move at random throughout some bounded habitat. Under this assumption, we model the interaction of the species via a system of partial differential equations of the form

$$(1.1)\qquad \begin{cases} u_{i_t} = \mu_i \Delta u_i + f_i(x, u_1, u_2) u_i & \text{in } \Omega \times (0, \infty), \\ B_i u_i = 0 & \text{on } \partial\Omega \times (0, \infty), \\ u_i(x, 0) = u_i^0(x) \geq 0 & \text{in } \Omega, \end{cases}$$

$i = 1, 2$, where u_i denotes the *population density* of the ith interacting species, $\Omega \subseteq \mathbf{R}^N$ (usually $N = 2$ or 3) is the habitat in question and

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