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HUMBERT SURFACES AND TRANSCENDENCE PROPERTIES OF AUTOMORPHIC FUNCTIONS

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Dedicated to Wolfgang M. Schmidt on the occasion of his 60th birthday

1. Introduction and statement of results. Let G be a Zariski connected reductive algebraic group defined over Q such that the abelian part of $G(\mathrm{R})$ is compact. Let $G^o(\mathrm{R})$ be the topological identity component and K be a maximal compact subgroup of $G^o(\mathrm{R})$. Suppose that the quotient $G^o(\mathrm{R})/K$ has an invariant complex structure and hence is isomorphic as a complex manifold to a bounded symmetric domain $\mathcal{D} \subset \mathrm{C}^m$ for an integer $m \geq 1$. A point $z \in \mathcal{D}$ is called a special point if it is the fixed point of a maximal torus $T \subset G$ defined over Q for which $T(\mathrm{R})$ is compact. Suppose that $\mathcal{D}$ is realized in C^m in such a way that the special points are in $\mathcal{D} \cap \overline{\mathrm{Q}}^m$. If Γ is a (neat) arithmetic subgroup of G, there is a Γ-invariant holomorphic map $J = J(\mathcal{D}, \Gamma)$ of $\mathcal{D}$ into a projective space which induces a biregular isomorphism of $\Gamma\backslash\mathcal{D}$ onto a complex quasi-projective variety V [**1**]. Moreover, Faltings showed in [**9**] that the variety V can be defined over $\overline{\mathrm{Q}}$ and that the $\overline{\mathrm{Q}}$-structure of V may be uniquely determined by requiring of $(\mathcal{D}, J, V)$ that all special points $z \in \mathcal{D}$ have $\overline{\mathrm{Q}}$-rational image point $J(z)$ in V. (For modulus varieties of abelian varieties of given PEL-type this was shown in [**22, 23**]. Faltings' approach of course bypasses abelian varieties. For Hilbert modular surfaces, Faltings' proof is written out in [**25**, p. 82]. We call a triple $(\mathcal{D}, J, V)$ as above a normalized model over $\overline{\mathrm{Q}}$ for (G, Γ). It seems reasonable to make the following:

Prediction. *Let $(\mathcal{D}, J, V)$ be a normalized model over $\overline{\mathrm{Q}}$ for (G, Γ) with $\mathcal{D} \subset \mathrm{C}^m$. Then $z \in \mathcal{D} \cap \overline{\mathrm{Q}}^m$ and $J(z) \in V(\overline{\mathrm{Q}})$ if and only if z is a special point.*

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