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COMMUTATIVE ALGEBRAIC GROUPS AND REFINEMENTS OF THE GELFOND-FELDMAN MEASURE

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ABSTRACT. The main theorem of this paper is a measure of algebraic independence for numbers associated with a one-parameter subgroup of a commutative algebraic group defined over a number field. Qualitative results in this setting have been given by M. Waldschmidt, R. Tubbs and M. Ably, who provided measures as well. We refine Ably's quantitative results, separating the degree and the height in the limit case when the group contains a copy of the additive group of complex numbers, i.e., $\mathbf{G}_a$. This new results provides several interesting corollaries, in particular, a generalization of G. Diaz's refined Gelfond-Feldman measure to higher dimensions and an improvement of Tubbs' elliptic Gelfond-Feldman measure.

1. Introduction and statement of result. We begin with a review of the standard objects in this general setting. Although our presentation is slightly different, this is essentially the setting of [**1**] or [**43**]. Let G be a commutative algebraic group of dimension $d \geq 1$ defined over a number field K. Let $\mathbf{G}_a$ denote the additive group of complex numbers and $\mathbf{G}_m$ the multiplicative group of complex numbers. We assume that G decomposes as

$$G = \mathbf{G}_a^{d_0} \times \mathbf{G}_m^{d_1} \times G_2$$

with $d_0 \in \{0,1\}$, $d_1 \geq 0$ and G_2 a commutative algebraic group of dimension $d_2 = d - d_0 - d_1$, defined over K, and with no linear factor.

We let $\phi : \mathbf{C} \to G(\mathbf{C})$ be a one-parameter subgroup, i.e., an analytic homomorphism whose image is Zariski dense in $G(\mathbf{C})$. Given complex

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