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SMALL SOLUTIONS TO SYSTEMS OF LINEAR CONGRUENCES OVER NUMBER FIELDS

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Dedicated to Professor Wolfgang M. Schmidt on his 60th birthday

1. Introduction. There has been much activity, in recent years, in developing theorems from the classical geometry of numbers into the adèle space of a number field. The pioneering work in this area was done independently by McFeat [**11**] and Bombieri and Vaaler [**3**], who produced the adelic analogue of Minkowski's successive minima theorem. Such machinery has primarily been used to solve various diophantine problems. The adèle space approach has been amply justified by relatively simple inequalities, and a much clearer presentation of the roles played by arithmetic and the geometry of Euclidean space. In fact, arithmetic is the result of geometry at the nonarchimedean completions of the number field. Here we wish to illustrate these methods by producing new sharp upper bounds for the size of solutions to systems of linear congruences over number fields. In the early 1900's, Aubry [**1**] and Thue [**13**] independently proved the following result which is now often referred to as the Aubry-Thue theorem.

Theorem 1. *Let a, b and $m > 0$ be integers. Then there exists an integer solution to*

$$ax + by \equiv 0 \pmod{m}$$

where $0 < \max\{|x|, |y|\} \le m^{1/2}$.

Improvements and generalizations in various directions were given by Vinogradov [**14**], De Backer [**9**], Ballieu [**2**] and Nagell [**12**]. In 1951 Brauer and Reynolds [**4**] used Dirichlet's box principle to prove the following extension of the Aubry-Thue theorem.

Theorem 2. *Let A be an $M \times N$ matrix having rational integer*

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