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THE SPECTRAL THEORY OF SECOND ORDER TWO-POINT DIFFERENTIAL OPERATORS IV. THE ASSOCIATED PROJECTIONS AND THE SUBSPACE $\mathcal{S}_\infty(L)$

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ABSTRACT. This paper is the final part in a four-part series on the spectral theory of a two-point differential operator L in $L^2[0,1]$, where L is determined by a formal differential operator $l = -D^2 + q$ and by independent boundary values B_1, B_2. For the family of projections $\{Q_{0k}\}_{k=1}^n \cup \{Q'_k\}_{k=k_0}^\infty \cup \{Q''_k\}_{k=k_0}^\infty$ which map $L^2[0,1]$ onto the generalized eigenspaces of L, it is determined whether or not the family of all finite sums of these projections is uniformly bounded in norm. Equivalently, for the subspace $\mathcal{S}_\infty(L)$ consisting of all $u \in L^2[0,1]$ with $u = \sum_{k=1}^n Q_{0k}u + \sum_{k=k_0}^\infty Q'_k u + \sum_{k=k_0}^\infty Q''_k u$, it is determined whether or not $\mathcal{S}_\infty(L) = \overline{\mathcal{S}_\infty(L)} = L^2[0,1]$. It is necessary to modify the projections and $\mathcal{S}_\infty(L)$ in the multiple eigenvalue case.

1. Introduction. In this paper we conclude our four-part series on the spectral theory of a linear second order two-point differential operator L in the complex Hilbert space $L^2[0,1]$. Let L be the differential operator in $L^2[0,1]$ defined by

$$\mathcal{D}(L) = \{u \in H^2[0,1] \mid B_i(u) = 0,\ i = 1,2\},$$
$$Lu = lu,$$

where

$$l = -\left(\frac{d}{dt}\right)^2 + q(t)\left(\frac{d}{dt}\right)^0$$

is a second order formal differential operator on the interval $[0,1]$ with $q \in C[0,1]$, B_1, B_2 are linearly independent boundary values given by

$$B_1(u) = a_1u'(0) + b_1u'(1) + a_0u(0) + b_0u(1),$$
$$B_2(u) = c_1u'(0) + d_1u'(1) + c_0u(0) + d_0u(1),$$

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