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# QUASICONFORMAL MAPS AND POINCARÉ DOMAINS

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ABSTRACT. In this paper we examine the invariance of Poincaré domains under quasi-conformal maps which satisfy a global integrability condition on the Jacobian. We show directly that, under such a map, the image of a John domain or a domain satisfying a quasi-hyperbolic boundary condition is $p$-Poincaré for $p \geq p_0$, here $p_0$ depends on the quasi-conformal map. Corresponding results are provided by $(q,p)$-Poincaré domains. We also provide sufficient conditions for which "rooms and corridors" domains are mapped to Poincaré domains.

**1. Introduction.** In this paper we examine the invariance of Poincaré domains under quasi-conformal maps which satisfy a global integrability condition on the Jacobian. Sufficient geometric conditions for Poincaré domains abound. Here we consider certain classes of Poincaré domains which include John domains, domains satisfying a quasi-hyperbolic boundary condition, and "rooms and corridors domains."

Hurri [**13**] has shown that Poincaré domains are invariant under locally bi-Lipschitz maps. However, one can see from the Riemann mapping theorem alone that quasi-conformal maps need not preserve Poincaré domains. In fact, a simply connected domain formed by adjoining a sequence of "rooms and corridors" domains [**13**, Example 5.2] to the unit ball can be constructed so that it is not Poincaré for any exponent $p$. To ensure that the image domain is Poincaré, we utilize an integrability condition introduced by Astala and Koskela [**1**]. Namely, letting $D$ and $D^*$ be domains in $\mathbf{R}^n$ and $f : D \to D^*$ a quasi-conformal

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