ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 26, Number 4, Fall 1996

APPLICATION OF CATLIN'S BOX CONSTRUCTION TO SUBELLIPTICITY OF $n-1$ FORMS

LOP-HING HO

1. Introduction. Subelliptic estimates for the $\bar{\partial}$-Neumann problem is the main tool in establishing local regularity for the solution of the $\bar{\partial}$-Neumann problem, which has lots of applications in several complex variables. On strictly pseudoconvex domains, subelliptic estimates hold with $\varepsilon = 1/2$. The question is much harder on weakly pseudoconvex domains. The most outstanding works on weakly pseudoconvex domains are by Kohn [**12**], Catlin [**1**] and [**2**]. Catlin used D'Angelo's [**4**] notion of finite type to establish necessary and sufficient conditions for subellipticity. Catlin [**3**] constructed a plurisubharmonic function to prove that in $\mathbf{C}^2$ if a point $z_0 \in b\Omega$ is of finite type m, then a precise subelliptic estimate of order $\varepsilon = 1/m$ holds at z_0. This result is established earlier in Kohn [**12**] and also in Fornaess and Sibony [**7**].

In this note we establish a result similar to the above-mentioned theorem in $\mathbf{C}^2$. We will prove a result for $(p, n-1)$ forms for non-pseudoconvex domains in $\mathbf{C}^n$. Some previous results in non-pseudoconvex domains are done in Hörmander [**11**], Ho [**8, 9**] and [**10**]. Hörmander dealt mainly with the case where the Levi-form is nondegenerate. Ho dealt with the case that the Levi-form of a vector field is bounded below by a certain nonnegative function. Here we deal with the case where there is a vector field whose Levi-form is nonnegative and with 'finite type' m, then we show that a subelliptic estimate of order $\varepsilon = 1/m$ holds for $(p, n-1)$ forms. This is essentially the best result we can expect with the existence of such a single vector field. (See Catlin [**1**] and Ho [**9**].) We use extensively the 'box construction' and the method of constructing plurisubharmonic functions in Catlin [**3**].

Though it seems that the theorem is not a surprising one, we should note that the question of subelliptic estimate is much more subtle for non-pseudoconvex domains. We given an example in the last part

Received by the editors on June 7, 1994, and in revised form on January 24, 1995.