ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 26, Number 4, Fall 1996

NORM INEQUALITIES WITH POWER WEIGHTS FOR HÖRMANDER TYPE MULTIPLIERS

RANDY COMBS

1. Introduction. Let $m(x)$ be a bounded, measurable function on $\mathbf{R}^n$. The operator $T_m f$ defined by the Fourier transform equation

$$(T_m f)^\wedge(x) = m(x)\hat{f}(x)$$

is called a multiplier operator with multiplier m. Denote by λ a nonnegative real number, s a number greater than or equal to 1, $|x| \sim R$ the annulus $\{x : R < |x| < 2R\}$, and $\alpha = (\alpha_1, \dots, \alpha_n)$ a multi-index of nonnegative integers α_j with norm $|\alpha| = \alpha_1 + \cdots + \alpha_n$. We say $m \in M(s, \lambda)$ if

(1)
$$B(m,s,\lambda) = ||m||_\infty + \sup_{R>0,|\alpha|\le\lambda} \left(R^{s|\alpha|-n} \int_{|x|\sim R} |D^\alpha m(x)|^s \, dx \right)^{1/s} < \infty$$

when λ is a positive integer. For the case where λ is not an integer, let 1 be the integer part of λ and let $\gamma = \lambda - l$. We say $m \in M(s, \lambda)$ if

(2)
$$B(m,s,\lambda) = B(m,s,l) + \sup_{R>0, 0<|z|<R/2} I(R,z) < \infty$$

where

$$I(R,z) = \sup_{|\alpha|=l} \left((R/|z|)^{\gamma s} R^{s|\alpha|-n} \times \int_{|x|\sim R} |D^\alpha m(x) - D^\alpha m(x-z)|^s \, dx \right)^{1/s}.$$

If λ is an integer, then those multipliers belonging to $M(2, \lambda)$ are the classical Hörmander-Mikhlin multipliers. The definition given here appears in [**4**].

Received by the editors on August 17, 1993, and in revised form on May 2, 1994.