ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 27, Number 1, Winter 1997

ON CHARACTERIZATION OF STRONGLY EXTREME POINTS IN KÖTHE-BOCHNER SPACES

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ABSTRACT. It is shown that the necessity in the characterization of strongly extreme points in Köthe-Bochner space $E(X)$, given by H. Hudzik and M. Mastyło in [**2**], is true without requiring that E be (LUR) and X be separable. The corollary concerning strongly extreme points in Musielak-Orlicz spaces of Bochner type is presented.

1. Introduction. Denote by $\mathbf{N}$ and $\mathbf{R}$ the sets of natural and real numbers, respectively. Let (T, Σ, μ) denote a measure space with a σ-finite and complete measure μ and $L^0 = L^0(T)$ the space of μ-equivalence classes of Σ-measurable real-valued functions. The notation $f \le g$ for $f, g \in L^0$ will mean that $f(t) \le g(t)$ μ-almost everywhere in T.

A Banach space $(E, \|\cdot\|_E) \subset L^0$ is said to be a Köthe space if

(i) $|f| \le |g|$, $f \in L^0$, $g \in E$ imply $f \in E$ and $\|f\|_E \le \|g\|_E$;

(ii) $\operatorname{supp} E =: \cup\{\operatorname{supp} f : f \in E\} = T$.

Now let us define the type of spaces to be considered in this paper. For a real Banach space $(X, \|\cdot\|_X)$, denote by $\mathcal{M}(T, X)$, or just $\mathcal{M}(X)$, the family of all strongly measurable functions $f : T \to X$ identifying functions which are μ-almost everywhere equal. Let

$$E(X) = \{f \in \mathcal{M}(X) : \|f(\cdot)\|_X \in E\}.$$

Then $E(X)$ becomes a Banach space with the norm

$$\|f\| = \|\|f(\cdot)\|_X\|_E,$$

and it is called a *Köthe-Bochner space.*

Received by the editors on June 10, 1994.
1991 Mathematics Subject Classification. 46E30, 46E40, 46B20.
Key words and phrases. Köthe-Bochner space, strongly extreme points, local uniform rotundity, Musielak-Orlicz space.
Supported by KBN grant 2 P03A 031 10.