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SOME ISOMORPHIC PREDUALS OF ℓ_1 WHICH ARE ISOMORPHIC TO c_0

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ABSTRACT. We introduce property (FS), which asserts that a Banach space has many c_0-"sub-decompositions," and show that if X is a Banach space with property (FS) and X^* is isomorphic to ℓ_1, then X itself is isomorphic to c_0.

1. Introduction. For quite some time it has been of interest to find out under what circumstances a separable $\mathcal{L}_\infty$-space is isomorphic to the simplest example of an $\mathcal{L}_\infty$-space, namely c_0.

Ghoussoub and Johnson [**3**] showed that a separable $\mathcal{L}_\infty$-space, which embeds into an order continuous Banach lattice, is isomorphic to c_0. (This result is based on earlier results by Johnson and Zippin [**6**] and Rosenthal [**9**].)

Using isometric methods, Godenfroy and Li [**5**] showed that a separable $\mathcal{L}_\infty$-space, which can be renormed into an M-ideal in its bidual, is also isomorphic to c_0.

Godefroy, Kalton and Saphar [**4**, Proposition 7.8] showed that c_0 is the only isometric predual of ℓ_1, which is a u-ideal.

In [**5**] the authors pose the question whether an isometric predual of ℓ_1, which has property (u), is isomorphic to c_0. We give a partial result in this direction:

Definition 1. A separable Banach space X has *property* (FS), if every shrinking finite-dimensional decomposition (F_n) of X has the following property: Every increasing sequence (m_n) of positive integers has a further subsequence (k_n), so that (F_{k_n}) is a c_0-decomposition for its closed linear span $[F_{k_n}]$.

We call a sequence (F_n) of subspaces of a Banach space X a c_0-*decomposition for its closed linear span*, if it satisfies the following:

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