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FACTORIZATIONS IN UNIVERSAL OPERATOR SPACES AND ALGEBRAS

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ABSTRACT. We build on previous work with V. Paulsen, using a notion of quantum variables. This enables us to give an explicit description of the norm in many universal C^*-algebras, operator algebras and operator spaces. This yields curious factorization results, for example a "generalized Fourier series" representation of all continuous functions on a compact group.

1. Introduction. Let f be a function in the Wiener algebra. Thus, we think of f as a function on the circle, and we may write $f = \sum_{k=-\infty}^{\infty} a_k e^{ik\theta}$, where $\sum_{k=-\infty}^{\infty} |a_k| < \infty$. Now rewrite f in a slightly different but exactly equivalent way:

$$f = [\, b_1 \quad b_2 \quad b_3 \quad \cdots \,] \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & e^{i\theta} & 0 & 0 & \cdots \\ 0 & 0 & e^{-i\theta} & 0 & \cdots \\ 0 & 0 & 0 & e^{2i\theta} & \cdots \\ \vdots & \vdots & & & \ddots \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \end{bmatrix}.$$

This may be done, for example, by relabelling the double series as an ordinary series $d_1 + d_2 e^{i\theta} + d_3 e^{-i\theta} + d_4 e^{2i\theta} + \cdots$, and then taking the $b_k = |d_k|^{1/2}$ and c_k equal to b_k multiplied by a complex number of modulus 1. Then the row and column matrices above have bounded norm. Let us write the factorization above as $f = \underline{\mathrm{b}}^t Z(\theta) \underline{\mathrm{c}}$. The coefficients ($b_k$ and c_k) are no longer uniquely determined; however, the Wiener algebra norm may be obtained by

$$\|f\|_W = \sum_{k=-\infty}^{\infty} |a_k| = \min\{\|\underline{\mathrm{b}}^t\| \, \|\underline{\mathrm{c}}\| : f = \underline{\mathrm{b}}^t Z(\theta) \underline{\mathrm{c}}\}.$$

We can again rewrite f equivalently as $f = \underline{\mathrm{b}}^t Z(\theta) \underline{\mathrm{c}}$, but now we allow $Z(\theta)$ to be a diagonal matrix with powers $e^{i\bar{k}\theta}$ on the diagonal in any

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