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MATRIX INNER PRODUCT HAVING A MATRIX SYMMETRIC SECOND ORDER DIFFERENTIAL OPERATOR

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ABSTRACT. In this note we characterize those positive definite matrices of measures whose matricial inner product has a symmetric left-hand side matrix second order differential operator.

1. Introduction. Let W be an $N \times N$ positive definite matrix of measures (i.e., for any Borel set $A \subset \mathbf{R}$, $W(A)$ is a semi-definite positive numerical matrix). We put $\langle\ ,\ \rangle_W$ for the matrix inner product defined by W:

$$\langle P, Q\rangle_W = \int_{\mathbf{R}} P(t)\, dW(t) Q^*(t) \in M_{N\times N}, \qquad P, Q \in \mathbf{P}_{N\times N}, \tag{1.1}$$

where we denote by $M_{N\times N}$ the space of numerical $N \times N$ matrices, by $\mathbf{P}_{N\times N}$ the space of $N \times N$ matrix polynomials, and by Q^* the Hermitian adjoint of Q.

Orthogonal matrix polynomials on the real line with respect to a positive definite matrix of measures have been considered in detail in M.G. Krein [**12**] or, more recently, by Aptekarev and Nikishin [**1**], Geronimo [**7**], Sinap and Van Assche [**14**], Duran and Van Assche [**6**], Duran and Lopez-Rodriguez [**5**] and Duran [**3, 4**]. In [**3, 6**], a very close relationship between orthogonal matrix polynomials and scalar polynomials satisfying a higher order recurrence relation has been established. However, as far as the author knows, no general results concerning orthogonal matrix polynomials and differential equations are known (for some examples of orthogonal matrix polynomials satisfying differential equations, see [**9, 10**].

In this note we characterize those positive definite matrices of measures whose matrix inner product, defined as before, has a symmetric

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