ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 27, Number 2, Spring 1997

MODULE TYPES

JOHN DAUNS

ABSTRACT. The direct sum decomposition of torsion free, i.e., nonsingular modules, into direct summands belonging to saturated classes of modules is developed here for the first time for arbitrary not necessarily torsion free modules. The previous classification of modules into types I, II, III, molecular, continuous molecular, and bottomless is extended to all modules from the previous torsion-free case. It is shown that there exists a contravariant functor Σ applicable to any associated ring R with identity—where $\Sigma(R)$ is a complete Boolean lattice. Each element $\Delta \in \Sigma(R)$ is a saturated class of right R-modules. The above six classes of modules are special examples of a general phenomenon—a universal saturated class. These have various functorial properties connected with the functor Σ. It is shown that there is a class of pairwise disjoint universal saturated classes one for each cardinal number.

0. Introduction. A saturated class of unital modules Δ over some ring R is a class of modules closed under isomorphic copies, submodules, direct sums, and injective hulls. In previous studies by Goodearl and Boyle [**23**], Rios and Tapia [**36**] and the author [**13, 14, 15**] the modules were required in addition to be torsion-free, that is, nonsingular. One of the objectives of this article will be to extend presently existing theory for torsion-free modules this theory to arbitrary modules including torsion and mixed. There are (infinitely) many examples. In fact, it is shown in this article that, for a fixed ring R and any given nonempty class of R-modules Υ, this class Υ generates a clearly describable unique saturated class (Proposition 2.5).

The saturated classes used here are special cases of the more general Wisbauer classes $\sigma[M]$ [**38**] as well as the natural classes used very recently by Page and Zhou [**33, 34, 35, 39**]. On the other hand, various special cases of torsion-free saturated classes have been used by different authors, for a long time, in different contexts without abstractly formalizing this concept [**32, 28, 23, 22, 3, 7, 8, 9**].

Received by the editors on April 28, 1995, and in revised form on September 11, 1995.
1991 AMS *Subject Classifications.* 16D50, 16E50, 16D70.