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ALGEBRAIC PROPERTIES OF THE LIAPUNOV AND PERIOD CONSTANTS

ANNA CIMA, ARMENGOL GASULL,
VÍCTOR MAÑOSA AND FRANCESC MAÑOSAS

ABSTRACT. We give several algebraic properties of the Liapunov and period constants that simplify their effective computation. We apply them to get the first Liapunov and period constants and the second Liapunov constant for an arbitrary analytic system. Finally we apply them to some particular families of differential equations.

1. Introduction. Consider the differential equation:

$$\begin{aligned} \dot{x} &= -y + f(x,y), \\ \dot{y} &= x + g(x,y), \end{aligned} \tag{1}$$

where f and g are analytic functions in a neighborhood of $(0,0)$ and begin, at least, with second order terms. It is well known that the problem to determine if (1) has a center or a focus at the origin can be reduced to the study of the Poincaré return map, or equivalently to the computation of infinitely many real numbers, v_{2m+1}, $m \geq 1$, called the Liapunov constants. In fact, we have that if for some k, $v_3 = v_5 = \cdots = v_{2k-1} = 0$ and $v_{2k+1} \neq 0$ the origin is a focus, while if all v_{2m+1} are zero the origin is a center, see for instance [**1**].

A closely related problem is the following: Assume that (1) has a center, and consider the period of all its periodic orbits. When is this period independent of the orbit, or, in other words, when the center is isochronous? It turns out that the solution to this problem follows by computing infinitely many real numbers, T_{2m}, $m \geq 1$, called the *period constants* and by imposing that all of them vanish.

In [**5**] the authors give a survey of different ways to compute the Liapunov constants. From there it is clear that all the approaches involve a lot of computations. This implies that even with powerful

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