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QUALITATIVE ANALYSIS OF A SINGULARLY-PERTURBED SYSTEM OF DIFFERENTIAL EQUATIONS RELATED TO THE VAN DER POL EQUATIONS

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ABSTRACT. A method to qualitatively analyze certain three-dimensional singularly-perturbed, nonautonomous, nonlinear systems is presented. The analysis involves the construction of a trapping region for solutions of the system. This method can be applied to the Oregonator model of the Belousov-Zhabotinskii reaction. One result is a new and clearer proof of Hastings and Murray's result that there is a nontrivial periodic solution of the model.

0. Introduction. This paper is concerned with the singularly-perturbed, nonautonomous, nonlinear ordinary differential equation:

$$\begin{aligned} \dot{x} &= \frac{1}{\varepsilon}(y - xz) + e_1(t) \\ \dot{y} &= -x + e_2(t) \qquad (1) \\ \dot{z} &= \frac{1}{\varepsilon}(x^2/3 - 1 - z) + e_3(t) \end{aligned}$$

where $0 < \varepsilon \ll 1$, $e_i(t)$, $i = 1, 2, 3$ are bounded functions (for example, periodic functions with common period L), and with $e_2(t)$ small.

1. Motivation. There are certain muscle fibers in the heart known as the cardiac Purkinje fibers. The primary function of the Purkinje fibers is to transmit electrical pacemaker impulses in the heart. The Purkinje fiber also exhibits a secondary activity; if the fiber is not subject to any outside stimulus, then it spontaneously and regularly generates an electrical impulse. Noble derived a four-dimensional autonomous system of singularly-perturbed differential equations to model the Purkinje fiber by modifying the Hodgkin-Huxley equations [**9**].

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