ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 27, Number 3, Summer 1997

PHASE PORTRAITS OF QUADRATIC SYSTEMS WITH FINITE MULTIPLICITY THREE AND A DEGENERATE CRITICAL POINT AT INFINITY

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ABSTRACT. In this paper a quadratic system is meant to be the autonomous system of ordinary differential equations

$$\begin{aligned}\dot{x} &= a_{00} + a_{10}x + a_{01}y + a_{20}x^2 + a_{11}xy + a_{02}y^2 \\ &\equiv P(x,y), \\ \dot{y} &= b_{00} + b_{10}x + b_{01}y + b_{20}x^2 + b_{11}xy + b_{02}y^2 \\ &\equiv Q(x,y),\end{aligned}$$

where $\cdot$ is defined to be d/dt and $a_{ij}, b_{ij} \in R$, and $P(x,y)$ and $Q(x,y)$ are relatively prime real polynomials, of degree at most two, which are not both linear. We study the class of quadratic systems with finite multiplicity three, consisting of systems with three elementary critical points, possibly complex or coinciding, and a degenerate type of critical point at infinity, being a point, which upon bifurcation such that only elementary critical points result, leaves one critical point in the finite part of the plane and two or three critical points at infinity. The notation for such a degenerate infinite critical point is $M_{1,2}^i$ and $M_{1,3}^i$, respectively.

A system with an $M_{1,3}^i$ point can be represented by the system

$$\begin{aligned}\dot{x} &= \lambda x + \mu y + xy + y^2, \\ \dot{y} &= x + y^2,\end{aligned}$$

where $\lambda, \mu \in \mathbf{R}$, and 22 topologically different phase portraits are obtained.

A system with an $M_{1,2}^i$ point can be represented by the system

$$\begin{aligned}\dot{x} &= \lambda x + \mu y + \gamma xy + \delta(x + y^2), \\ \dot{y} &= x + y^2,\end{aligned}$$

where $\mu, \gamma, \delta \in \mathbf{R}$ whereas $\lambda \in \{0, 1\}$ and $\gamma \notin \{0, 1\}$. As a result of the classification, 119 topologically different phase portraits are obtained.

Received by the editors on January 24, 1995, and in revised form on April 20, 1995.
Key words and phrases. Quadratic systems, finite multiplicity 3, phase portraits.