

ON QUASI-POSITIVE DEFINITE FUNCTIONS
AND REPRESENTATIONS OF
HYPERGROUPS IN QP_n SPACES

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ABSTRACT. The purpose of this paper is to transfer the results about the relation between quasi-positive definite functions on groups and semigroups and their representations in spaces with an indefinite metric to the case of hypergroups. This work may be considered to continue the studies concerning the generalizations of Godement's theory about cyclic unitary representations of locally compact groups in Hilbert spaces and positive definite functions [3].

0. Introduction. The relation between cyclic unitary representations of a topological group G in Pontryagin spaces and quasi-positive definite functions on G has been thoroughly investigated by K. Sakai [7]. In [1], C. Berg and Z. Sasvári have studied indefinite functions on semigroups and their relation to representations in spaces with an indefinite metric. The present paper starts from these two articles, which suggested that the same problem can be transferred to the case of hypergroups. The main difficulty of this program consists in defining quasi-positive functions on hypergroups such that this definition becomes compatible with the ones for groups and semigroups. In addition, one should be able to relate them to representations of hypergroups in indefinite spaces. In Section 1 we define quasi-positive functions on hypergroups and we construct the indefinite space (and finally the Pontryagin space) associated with a quasi-positive function. In Section 2 we relate such functions to representations in indefinite spaces. In the last section we give examples of quasi-positive functions by starting with the study of functions of finite rank.

Hypergroups are locally compact Hausdorff spaces whose regular complex-valued Borel measures form an algebra, which has similar properties as the convolution algebra $(\mathcal{M}(G), *)$ of a topological group

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