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A CONVERGENCE RESULT FOR REGULAR BLASCHKE FRACTIONS

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1. Introduction. For a given α, $0 < |\alpha| < 1$, we shall call

$$\zeta(z) = \frac{\bar{\alpha}}{|\alpha|}\frac{\alpha - z}{1 - \bar{\alpha}z} \tag{1.1}$$

a *Blaschke factor*. Although $\zeta(z)$ is meaningful on the whole Riemann sphere (analytic except for a pole of order 1 at $z = 1/\bar{\alpha}$) it is often assumed that $|z| < 1$, in which case also $|\zeta(z)| < 1$, or even more: The function $z \to \zeta(z)$ maps the open unit disk D one to one onto itself. In the literature Blaschke factors are sometimes defined slightly differently from (1.1). A consequence of the normalizing in (1.1) is that

$$\zeta(0) = |\alpha| > 0, \tag{1.2}$$

which, in particular for *Blaschke products*,

$$B_n(z) = \prod_{k=1}^{n} \zeta_k(z) = \prod_{k=1}^{n} \frac{\bar{\alpha}_k}{|\alpha_k|}\left(\frac{\alpha_k - z}{1 - \bar{\alpha}_k z}\right), \tag{1.3}$$

is of advantage.

Sometimes it is convenient to accept also $\alpha = 0$ or $|\alpha| = 1$ or both, although in both cases something gets lost.

For $\alpha = 0$ one sometimes makes the convention that

$$\zeta(z) = z.$$

We shall here, unless otherwise stated, follow this convention. This can be obtained by taking $\bar{\alpha}/|\alpha|$ to be -1 if $\alpha = 0$. What here gets lost is seen as follows. Take

$$\alpha = \rho e^{i\theta}.$$

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