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CONSTRUCTION OF THE SOLUTIONS OF DIFFERENCE EQUATIONS IN THE FIELD OF MIKUSIŃSKI OPERATORS

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ABSTRACT. We construct the solutions of certain difference equations with variable coefficients in the field of Mikusiński operators $\mathcal{F}$. The method we are using is very similar to the method used for the difference equations with variable numerical coefficients. We analyze the character of the solutions of the difference equation obtained by using this method.

The considered difference equations can be treated as the discrete analogues for the differential equations whose coefficients are operator functions in the field $\mathcal{F}$. Therefore the obtained solutions can be treated as the approximate solutions for the corresponding differential equations.

1. Introduction. The set of continuous functions $\mathcal{C}_+$ with supports in $[0,\infty)$, with the usual addition and the multiplication given by the convolution

$$f(t) * g(t) = \int_0^t f(\tau)g(t-\tau)\,d\tau, \quad t > 0,$$

is a ring. By the Titchmarsh theorem, $\mathcal{C}_+$ has no divisors of zero, hence its quotient field can be defined (see [**2**], and, for more advanced topics, [**3**]). The elements of this field, the Mikusiński operator field $\mathcal{F}$, are called *operators*. They are quotients of the form

$$\frac{f}{g}, \quad f \in \mathcal{C}_+, \quad 0 \not\equiv g \in \mathcal{C}_+,$$

where the last division is observed in the sense of convolution. Every continuous function $a = a(t)$, $t \geq 0$, defines a unique operator which we denote simply by a. In that case, we write

$$a = \{a(t)\}.$$

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