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AN EXAMPLE OF DUAL CONTROL

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The C^*-algebras introduced by the author [**8**] to study index theorems on open manifolds are now known to be analytic counterparts of certain module categories appearing in controlled topology [**2**]. An apparently significant distinction is that the modules used in controlled topology are locally finite-dimensional, whereas the Hilbert spaces used to construct the C^*-algebras are locally infinite-dimensional. In the author's opinion, this distinction arises because the analysis of elliptic operators (which are the basic 'cycles' in analytic representations of K-homology) itself constitutes a form of 'control,' but in a 'spectral' rather than a 'spatial' direction. The purpose of this note is to reinforce this point of view by an example.

Let M be a compact odd-dimensional manifold, and let D be a generalized Dirac operator on M acting on a Hilbert space H of L^2 sections of the appropriate bundle. Then it is well known [**1, 5**] that D gives a cycle in Kasparov's analytic K-homology for M, and therefore gives a map

$$K^1(M) \longrightarrow \mathbf{Z}.$$

We will show how this map may be obtained using controlled C^*-algebra theory.

Recall [**8, 6**] that the basic object needed to define the C^*-algebra of a coarse space X is an X-module, that is, a Hilbert space equipped with an action of $C_0(X)$. Now we observe

Lemma 1. *The operator D endows H with the structure of an $|\mathbf{R}|$-module.*

(The notation $|\mathbf{R}|$ refers to the underlying coarse space of $\mathbf{R}$.) To see this we just use the spectral theorem, defining the action of $f \in C_0(\mathbf{R})$ on H by the operator $f(D)$. Observe that elliptic theory

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