ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 27, Number 4, Fall 1997

SOME GENERALIZATIONS OF UNIVERSAL MAPPINGS

M.M. MARSH

ABSTRACT. A mapping $f : X \to Y$ is *universal* if, for each mapping $g : X \to Y$, there is a point x in X such that $f(x) = g(x)$. We define several classes of mappings which properly contain the universal mappings and we establish relationships between these mappings, the fixed point property, the span of continua and the Borsuk-Ulam theorem.

1. Introduction, definitions and observations. In 1967, W. Holsztyński [**8**] defined universal mappings between topological spaces. A mapping $f : X \to Y$ is *universal* if, for each mapping $g : X \to Y$, there is a point x in X such that $f(x) = g(x)$. Holsztyński used this property of mappings to obtain several fixed point theorems. Others have also used this property to obtain fixed point results, e.g., see [**19, 18, 4, 15** and **16**].

In this paper we offer several generalizations of the universal property. We show that each of these properties has some utility in obtaining fixed point theorems. Furthermore, several of these properties have nice relationships to the notion of span of continua which was introduced by A. Lelek [**13**] in 1964. In Sections 2 through 4 we study these classes of mappings. We consider spaces which admit only certain of these mappings onto themselves, relationships to span and relationships to the fixed point property.

By a *continuum* we will mean a compact connected metric space. A continuous function will be referred to as a *map* or *mapping*. The following definitions are made for mappings between topological spaces although our interest will be primarily in mappings between continua. The definition of universal is given above.

A mapping $f : X \to Y$ is *weakly universal* if for each mapping $g : X \to X$ there is a point x in X such that $f(x) = fg(x)$. A

AMS (MOS) *Mathematics subject classification.* Primary 54H25, Secondary 54F20, 54F62.
Keywords and phrases. Universal maps, weakly universal maps, fixed point property, span of continua.
Received by the editors on October 25, 1990, and in revised form on January 15, 1996.