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LOCALIZED SOLUTIONS OF SUBLINEAR ELLIPTIC EQUATIONS: LOITERING AT THE HILLTOP

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ABSTRACT. We establish existence of infinitely many localized twice-differentiable radial solutions to the equation $\Delta v + f(v) = 0$ in $\mathbf{R}^N$, where f is linearly bounded above. Such equations govern the spatial profiles of solitary-wave solutions to nonlinear wave equations with global regularity of solutions. We use constructive methods to show that there are localized solutions with any prescribed number of nodes.

1. Introduction. We consider the semilinear wave equation

$$u_{tt} - \Delta u = g(u), \tag{1.1}$$

where solutions u are complex-valued functions on spacetime $\mathbf{R}^{N+1}$, with spatial dimension $N \geq 2$, and where the nonlinearity $g : \mathbf{C} \to \mathbf{C}$ has the property that $g(se^{i\psi}) = g(s)e^{i\psi}$ for all real s and ψ. Such a function g is determined by its restriction to the real axis, which is necessarily odd, and which we assume to be real. Let $G(s) \equiv \int_0^s g(s')\,ds'$ be the primitive of g. If $G(s) \leq 0$ for all real s, then conservation of the energy $\mathcal{E}[u, u_t] \equiv \int_{\mathbf{R}^N} \{(1/2)|u_t|^2 + (1/2)|\nabla u|^2 - G(|u|)\}\, d^N x$ implies, under growth conditions on g, that solutions to (1.1) with finite-energy initial data are bounded in bounded regions of spacetime [**8, 10**]. If, on the other hand, the primitive is positive at some amplitudes, then it is possible for singularities to develop. Here we will consider the well-behaved case in which $G(s) \leq 0$ for all s, consistent with global regularity of solutions.

We are interested in standing-wave solutions of the nonlinear wave equation, of the form $u(x,t) = e^{i\omega t}v(x)$, where ω is a real constant. For such a solution, the standing-wave profile $v : \mathbf{R}^N \to \mathbf{C}$ satisfies the associated nonlinear elliptic equation

$$\Delta v + f_\omega(v) = 0 \tag{1.2}$$

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