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SPECTRAL DOMAINS IN SEVERAL COMPLEX VARIABLES

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ABSTRACT. In this paper we study the concepts of spectral domain and complete spectral domain in several complex variables. For a domain Ω in $\mathbf{C}^n$ and an n-tuple T of commuting operators on a Hilbert space $\mathcal{H}$ such that the Taylor spectrum of T is a subset of Ω, we introduce the quantities $K_\Omega(T)$ and $M_\Omega(T)$. These quantities are related to the quantities $K_X(T)$ and $M_X(T)$ introduced by Paulsen for a compact subset X. When T is an n-tuple of 2×2 matrices, $K_\Omega(T)$ and $M_\Omega(T)$ are expressed in terms of the Carathéodory metric and the Möbius distance. This in turn answers a question by Paulsen for tuples of 2×2 matrices. We also establish von Neumann's inequality for an n-tuple of upper triangular Toeplitz matrices. We study the regularity of $K_\Omega(T)$ and $M_\Omega(T)$ and obtain various comparisons of these two quantities when T is an n-tuple of Jordan blocks.

1. Introduction. This work is motivated primarily by two papers, namely, [**1**] and [**23**]. The former shows a strong connection between operator theory and complex geometry by giving an operator theoretic proof of a fundamental result on invariant metrics for convex domains in $\mathbf{C}^n$. For an infinitesimal version of that result, see [**25**]. The second paper is a survey of results concerning spectral sets and centering around von Neumann's inequality.

In this paper we study the concepts of spectral domain and complete spectral domain in several complex variables. We use some ideas from complex geometry to obtain some results in multi-variable operator theory.

Our first group of results consists of improvements of results of several authors concerning n-tuples of 2×2 matrices. This is summarized in Theorem 1. As a consequence, we answer, in this case, a question of Paulsen, and we give a new proof of von Neumann's inequality for any n-tuple of 2×2 matrices.

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