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A SIGN-CHANGING SOLUTION FOR A SUPERLINEAR DIRICHLET PROBLEM

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ABSTRACT. We show that a superlinear boundary value problem has at least three nontrivial solutions. A pair are of one sign (positive and negative, respectively), and the third solution changes sign exactly once. The *critical level* of the sign-changing solution is bounded below by the sum of the two lesser levels of the one-sign solutions. If nondegenerate, the one sign solutions are of *Morse index* 1 and the sign-changing solution has Morse index 2. Our results extend and complement those of Z.Q. Wang [**12**].

1. Introduction. Let Ω be a smooth bounded region in $\mathbf{R}^N$, Δ the Laplacian operator, and $f \in C^1(\mathbf{R}, \mathbf{R})$ such that $f(0) = 0$. We assume that there exist constants $A > 0$ and $p \in (1, (N+2)/(N-2))$ such that $|f'(u)| \leq A(|u|^{p-1} + 1)$ for all $u \in \mathbf{R}$. Hence, f is *subcritical*, i.e., there exists $B > 0$ such that $|f(u)| \leq B(|u|^p + 1)$. Also, we assume that there exists $m \in (0, 1)$ such that

$$f(u)u - 2F(u) \geq muf(u), \tag{1}$$

where $F(u) = \int_0^u f(s)\, ds$, for all $u \in \mathbf{R}$. Finally, we make the assumption that f satisfies

$$f'(u) > \frac{f(u)}{u} \quad \text{for } u \neq 0 \quad \text{and} \quad \lim_{|u| \to \infty} \frac{f(u)}{u} = \infty, \tag{2}$$

i.e., f is *superlinear*. Let $0 < \lambda_1 < \lambda_2 \leq \lambda_3 \leq \cdots$ be the eigenvalues of $-\Delta$ with zero Dirichlet boundary condition in Ω. In this paper we study the boundary value problem

$$\begin{cases} \Delta u + f(u) = 0 & \text{in } \Omega \\ u = 0 & \text{in } \partial\Omega. \end{cases} \tag{3}$$

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