

# OSCILLATORY PROPERTIES OF THE SOLUTIONS OF IMPULSIVE DIFFERENTIAL EQUATIONS WITH A DEVIATING ARGUMENT AND NONCONSTANT COEFFICIENTS

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**ABSTRACT.** Sufficient conditions are found for oscillation of all solutions of the impulsive differential equation with a deviating argument

$$\begin{aligned}x'(t) + p(t)x(t - \tau) &= 0, \quad t \neq \tau_k, \\ \Delta x(\tau_k) &= b_k x(\tau_k), \quad t = \tau_k,\end{aligned}$$

where the function  $p$  is not of constant sign.

**1. Introduction.** In the last twenty years the number of investigations devoted to oscillatory and nonoscillatory behavior of solutions of functional differential equations has considerably increased. The greater part of the works on this subject published by 1977 are given in [4]. In the monographs [2] and [3] published respectively in 1987 and 1991, the oscillatory and asymptotic properties of the solutions of various classes of functional differential equations were systematically studied. The pioneer work devoted to the investigation of the oscillatory properties of the solutions of impulsive differential equations with a deviating argument was the work of Gopalsamy and Zhang [1]. In it the authors gave sufficient conditions for oscillation of the solutions of the impulsive differential equation with a deviating argument

$$(1) \quad \begin{aligned}x'(t) + p(t)x(t - \tau) &= 0, \quad \tau = \text{const}, \quad t \neq \tau_k; \\ \Delta x(\tau_k) &= x(\tau_k + 0) - x(\tau_k - 0) = b_k x(\tau_k - 0)\end{aligned}$$

where  $p$  is a nonnegative function. It is again there that conditions are given for the existence of nonoscillating solutions of the equation considered when  $p$  is a positive constant.

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