HEREDITARY PROPERTIES FOR DUALS OF
BOCHNER L_p-FUNCTION-SPACES

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ABSTRACT. For a finite and positive measure space (Ω, Σ, μ) hereditary results which hold for Bochner L_p-spaces are derived for their dual spaces. In addition, results of N. Kalton, G. Pisier and N. Randrianantoanina–E. Saab are given alternative proofs.

1. Introduction and preliminaries. There exists a long list on hereditary results for Bochner L_p-spaces, but virtually no equivalent statements are given for their dual spaces. The major problem is the lack of a satisfying representation of their dual spaces, respectively, meaning the second dual of the Bochner L_p-spaces. Let us mention some classical hereditary results concerning embedding classical sequence spaces. S. Kwapien showed that for $1 \leq p < \infty$, $c_0 \subset L_p(\mu, X)$, if and only if it embeds in X [12]. J. Mendoza proved that l_∞ can be found as a copy in $L_p(\mu, X)$, $1 \leq p < \infty$ if and only if it is isomorphically embedded in X, see [14]. Furthermore, the result that $l_1 \subset L_p(\mu, X)$ for $1 < p < \infty$ if and only if l_1 embeds in X, is due to G. Pisier [17]. We will show Pisier's result by different means, which allows the extension to the w^*-measurable case. In [21, p. 404] it was shown that some renorming properties of X are inherited by $L_p(\mu, X)$ for $1 \leq p < \infty$, the space of p-integrable vector-valued functions. M. Talagrand demonstrated in [22, p. 717] that weak sequential completeness of X passes to the function space $E(X)$, where E is an order continuous Banach lattice with a weak unit not containing c_0. N. Randrianantoanina and E. Saab obtained that $L_p(\mu, X)$, $1 < p < \infty$, enjoys the complete continuity property if and only if X does [18, p. 1111]. We will present an alternative proof and extend it to the dual of $L_p(\mu, X)$. The results are based on approximation results for operator-valued functions in Section 2, which may be of independent interest. In addition we extend results of J. Voigt [23], respectively the author [20], to the w^*-measurable case.