ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 28, Number 1, Spring 1998

ON PROPERTIES OF MULTIPLIERS OF CAUCHY TRANSFORMS

D.J. HALLENBECK AND K. SAMOTIJ

ABSTRACT. In this paper we prove that the lengths of images of certain rectifiable arcs under a multiplier f of fractional analytic Cauchy-Stieltjes transforms on the disk are uniformly bounded by a constant depending on the multiplier norm of f. As a consequence of this result, we also prove that $|f'(z)|^2$ is integrable with respect to area measure on every Stolz angle. Finally, we prove that our results are sharp in two different senses.

1. Introduction. Let $\Delta = \{z : |z| < 1\}$ and let $\Gamma = \{z : |z| = 1\}$. Let $\mathcal{M}$ denote the set of complex-valued Borel measures on Γ. For each $\alpha > 0$, let $\mathcal{F}_\alpha$ denote the family of functions f having the property that there exists a measure $\mu \in \mathcal{M}$ such that

$$f(z) = \int_\Gamma \frac{1}{(1 - \bar{x}z)^\alpha}\, d\mu(x) \tag{1}$$

for $|z| < 1$. In (1) and throughout this paper, each logarithm means the principal branch. $\mathcal{F}_\alpha$ is a Banach space with respect to the norm defined by

$$\|f\|_{\mathcal{F}_\alpha} = \inf\{\|\mu\|\} \tag{2}$$

where μ varies over all measures in $\mathcal{M}$ for which (1) holds and where $\|\mu\|$ denotes the total variation norm of μ. For $\alpha = 0$, let $\mathcal{F}_0$ denote the family of functions f having the property that there exists a measure $\mu \in \mathcal{M}$ such that

$$f(z) = f(0) + \int_\Gamma \log \frac{1}{(1 - \bar{x}z)}\, d\mu(x) \tag{3}$$

Received by the editors on August 17, 1995.
1991 AMS *Mathematical Subject Classification.* Primary 30E20, Secondary 30D99.
Key words and phrases. Multipliers, fractional Cauchy transform.