ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 28, Number 1, Spring 1998

OSCILLATION OF THE SOLUTIONS OF IMPULSIVE DIFFERENTIAL EQUATIONS AND INEQUALITIES WITH A RETARDED ARGUMENT

D.D. BAINOV, M.B. DIMITROVA AND A.B. DISHLIEV

ABSTRACT. Sufficient conditions for oscillation of all solutions of a class of impulsive differential equations and inequalities with a retarded argument and fixed moments of impulse effect are found.

1. Introduction. The impulsive differential equations are an adequate mathematical apparatus for simulation of processes and phenomena observed in control theory, physics, chemistry, population dynamics, biotechnologies, industrial robotics, economics, etc. Due to this reason, in recent years they have been an object of active research. In the monographs [**2**–**4**] a number of properties of their solutions are studied and an extensive bibliography is given.

We shall note that, in spite of the great number of investigations of the impulsive differential equations, their oscillation theory has not yet been elaborated unlike the oscillation theory of the differential equations with a deviating argument (see the monographs [**6**–**8**]).

The first work in which the oscillatory behavior of impulsive differential equations with a deviating argument and fixed moments of impulsive effect is investigated is [**5**]. Moreover, we shall note the work [**1**] in which the Sturmian theory for impulsive differential equations is considered.

In the present work sufficient conditions for oscillation of all solutions of a class of impulsive differential equations and inequalities with a retarded argument and fixed moments of impulse effect are found.

2. Preliminary notes. Let $\mathbf{N}_m = \{1, 2, \dots, m\}$ and h_i be positive constants, $i \in \mathbf{N}_m$, $\bar{h} = \max\{h_i : i \in \mathbf{N}_m\}$, $h = \min\{h_i : i \in \mathbf{N}_m\}$, $\{\tau_k\}_{k=1}^{\infty}$ be a monotone increasing, unbounded sequence of positive numbers, and let $\{b_k\}_{k=1}^{\infty}$ be a sequence of real numbers.

Received by the editors on March 28, 1995.