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ON THE EIGENVALUES OF BOUNDARY VALUE PROBLEMS FOR HIGHER ORDER DIFFERENCE EQUATIONS

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ABSTRACT. We consider the boundary value problem

$$\Delta^n y + \lambda Q(k, y, \Delta y, \dots, \Delta^{n-2} y) = \lambda P(k, y, \Delta y, \dots, \Delta^{n-1} y),$$
$$n \geq 2, \quad 0 \leq k \leq N,$$
$$\Delta^i y(0) = 0, \quad 0 \leq i \leq n-3,$$
$$\alpha \Delta^{n-2} y(0) - \beta \Delta^{n-1} y(0) = 0,$$
$$\gamma \Delta^{n-2} y(N+1) + \delta \Delta^{n-1} y(N+1) = 0$$

where $\lambda > 0$, α, β, γ and δ are constants satisfying $\alpha\gamma(N+1) + \alpha\delta + \beta\gamma > 0$, $\alpha, \gamma > 0, \beta \geq 0$ and $\delta \geq \gamma$. Upper and lower bounds for λ are established for the existence of positive solutions of this boundary value problem.

1. Introduction. Let a, b, $b > a$, be integers. We shall denote $[a, b] = \{a, a+1, \dots, b\}$. All other interval notation will carry its standard meaning, e.g., $[0, \infty)$ denotes the set of nonnegative real numbers. Also, the symbol Δ^i denotes the ith forward difference operator with stepsize 1.

In this paper we shall consider the nth order difference equation

$$\Delta^n y + \lambda Q(k, y, \Delta y, \dots, \Delta^{n-2} y) = \lambda P(k, y, \Delta y, \dots, \Delta^{n-1} y), \quad k \in [0, N] \tag{1.1}$$

and the boundary conditions

$$\Delta^i y(0) = 0, \quad 0 \leq i \leq n-3, \tag{1.2}$$
$$\alpha \Delta^{n-2} y(0) - \beta \Delta^{n-1} y(0) = 0, \tag{1.3}$$
$$\gamma \Delta^{n-2} y(N+1) + \delta \Delta^{n-1} y(N+1) = 0 \tag{1.4}$$

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