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RANDOM FOURIER SERIES AND ABSOLUTE SUMMABILITY

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ABSTRACT. In this article it is shown that many classical results concerning absolute summability of Fourier series can be obtained by random methods. An estimate on the modulus of continuity of random Fourier series is obtained. This estimate is then applied to obtain some new sufficient conditions for the absolute summability of the Fourier series.

1. Introduction. A classical result of Bernstein [**1**] states that if f is a periodic Lipschitz function of order greater than $1/2$, then its Fourier series converges absolutely. Later Hyslop [**3**] extended Bernstein's theorem by using absolute Cesaro summability, see Section 3 for the definition of absolute summability. Hyslop proved that if f is a Lipschitz function of order α and $0 < \alpha \leq 1/2$, then the Fourier series of f is summable $|C, \beta|$ whenever $\alpha + \beta > 1/2$. Hyslop's result was extended by McFadden [**9**] and Lal [**7**] to the case of summability for certain types of Nörlund means, see Section 3. There are a considerable number of sufficient conditions for the absolute summability and absolute convergence of Fourier series, see for example [**10**] and the references therein. Our contribution is that most of the sufficiency conditions for absolute summability are obtained by random methods using Khinchin's inequality and the modified versions of the three above-mentioned results, see for example Theorem 3.7 and Theorem 3.8. In some instances we obtain modest extensions of the classical results.

2. Random Fourier series. We consider the randomization of the Fourier series of f. Let

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