ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 28, Number 2, Summer 1998

ON THE PROFINITE TOPOLOGY OF THE AUTOMORPHISM GROUP OF A RESIDUALLY TORSION FREE NILPOTENT GROUP

E. RAPTIS AND D. VARSOS

ABSTRACT. Let G be a finitely generated residually torsion-free nilpotent group. Then every polycyclic-by-finite subgroup H of $\operatorname{Aut} G$ is closed in the profinite topology on $\operatorname{Aut} G$.

1. It is known that, if G is a finitely generated (f.g.) residually finite ($\mathcal{RF}$) group, then its automorphism group $\operatorname{Aut} G$ is also $\mathcal{RF}$, cf. [**2**]. This gives a motive to ask if the subgroup separability of a group implies the subgroup separability of its automorphism group.

Although the automorphism group of a free group is not subgroup separable, see Proposition 1 below, we prove that polycyclic-by-finite groups of automorphisms of a residually torsion-free nilpotent group G are closed in the profinite topology on $\operatorname{Aut} G$.

2. The profinite topology on a group G is the topology in which a base for the open sets is the set of all cosets of normal subgroups of finite index in G.

A group is said to be subgroup separable if all of its f.g. subgroups are closed in the profinite topology on G or, equivalently, if any pair of distinct finitely generated subgroups of G may be mapped to distinct subgroups in some finite quotient of G. Note that G is $\mathcal{RF}$ if and only if the trivial subgroup is closed in the profinite topology on G.

Proposition 1. *Let $F = \langle a, b\rangle$ be the free group of rank 2. There exists a subgroup of* $\operatorname{Aut} F$ *which is not closed in the profinite topology on* $\operatorname{Aut} F$.

Proof. The group K with presentation $K = \langle t, x, y | t^{-1}xt =$

Received by the editors on July 11, 1995, and in revised form on August 8, 1996.
1991 AMS *Mathematics Subject Classification.* 20F28, 20E26.