A SIMPLIFIED PROOF OF OSHIRO'S THEOREM
FOR co-H RINGS

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The purpose of this note is to give an alternative proof of the following result originally due to Oshiro [**7**]:

Theorem. *For a ring, R, the following are equivalent*:

(i) *R is a right co-H ring.*

(ii) *Every right R-module is the direct sum of a projective and a singular.*

(iii) *The class of projective right R-modules is closed under essential extensions.*

(iv) (a) *Every right R-module which is not singular has a nonzero projective direct summand.*

(b) *R has* ACC *on the right annihilators of subsets of R.*

Note. In [**7**] it was also shown that such a ring is semi-primary $QF-3$.

Definition 1. A module M is said to be CS if each submodule is essential in some direct summand of M, or equivalently, if every essentially closed submodule is a direct summand. A module M is said to be $\sum$-CS if every direct sum of copies of M is CS. Sometimes CS modules are called *extending* modules.

Definition 2 [**7**]. A ring R is said to be *right co-Harada*, abbreviated to *right co-H*, if every projective right module is CS. It is easy to see that R is right co-H if and only if R is $\sum$-CS as a right R-module.

In [**7**], the proof of the above theorem, though revealing much about the structure of such rings, was very long and, in places, rather difficult

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