ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 28, Number 2, Summer 1998

RIEMANNIAN MANIFOLDS WITH CONICAL SINGULARITIES

ZHONG-DONG LIU, ZHONGMIN SHEN AND DAGANG YANG

ABSTRACT. We study Riemannian manifolds with isolated conical singularities, in particular, the relationship between the curvature near singularities and the geometry of the tangent cones. We obtain some local and global rigidity theorems for singular metrics.

1. Introduction. Singular spaces appear naturally in many areas in both mathematics and physics. In general, it is difficult to study the global geometry of singular spaces. There is a special class of singular spaces, namely, Riemannian manifolds with conical singularities, which have been investigated by several people. See, e.g., [**3, 4, 7, 8, 15, 16**], etc. In this paper we shall study the geometric structure of singular Riemannian manifolds from a different point of view.

Throughout this paper, $\mathbf{S}^{n-1}$ and $\mathbf{B}^n(r)$ denote the standard unit sphere and the standard r-ball in the Euclidean space $\mathbf{R}^n$, respectively. Let Σ be an $(n-1)$-dimensional connected closed C^∞ manifold. The *topological cone* $C(\Sigma)$ over Σ is defined by

$$C(\Sigma) := [0,\infty)\times\Sigma/(\{0\}\times\Sigma).$$

Denote points in $C(\Sigma)$ by $[t,x]$ and the vertex by o. For $r>0$, put $C_r(\Sigma)=\{[t,x]\in C(\Sigma) : t<r\}$. Thus $\mathbf{B}^n(r)=C_r(\mathbf{S}^{n-1})$. From now on, "$=$" means the canonical pointed-isometry (preserving the vertices). A lens space is the quotient space $\mathbf{S}^{n-1}/\Gamma$, where Γ is a finite group acting freely on $\mathbf{S}^{n-1}$ by isometries. We shall always denote by $d\theta^2$ the canonical quotient metric on $\mathbf{S}^{n-1}/\Gamma$. The action of Γ on $\mathbf{S}^{n-1}$ can be lifted to an action of Γ on $\mathbf{B}^n(r)$ such that $\mathbf{B}^n(r)/\Gamma = C_r(\mathbf{S}^{n-1}/\Gamma)$. Thus $C_r(\mathbf{S}^{n-1}/\Gamma)$ is a topological orbifold.

An n-dimensional C^∞ *manifold with isolated conical singularities* is a Hausdorff space with countably many points, called singular

Received by the editors on January 28, 1996.
1991 AMS *Mathematics Subject Classification.* Primary 53C20.
The first author is supported in part by a grant from the University of South Carolina Research and Productive Scholarship Fund.