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ASYMPTOTIC CONSTANCY OF SOLUTIONS OF DELAY-DIFFERENTIAL EQUATIONS OF IMPLICIT TYPE

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ABSTRACT. We study delay-differential equations with time-state depending lag. We will prove that, under integrability conditions, any solution converges. Reciprocally, for any possible ξ, there exists a solution x such that $x(t) \to \xi$.

1. Introduction. Let b and τ be two positive reals, $I = [0, \infty)$ and $B_n[0, 2b]$ the closed ball centered at the origin with radius $b < \infty$, contained in $\mathbf{C}^n$.

Consider the functions f and r satisfying the following assumption C_1.

C_1) $f : I \times B_n[0, 2b] \to \mathbf{C}^n$ is a continuous function satisfying $|f(t, x)| \le \mu(t)|x|$ where $\mu : I \to I$ is continuous and $r : I \times B_n[0, b] \to [0, \tau]$ is another continuous function.

We are interested in the existence and asymptotic behavior of solutions of delay-differential equations of the type

$$\dot{x}(t) = f(t, x(t - r(t, x(t))) - x(t)). \tag{1}$$

Define, for $t \ge 0$,

$$m_b(t) = \sup_{|x| \le b} r(t, x) \quad \text{and} \quad \lambda_b(t) = \mu(t) \cdot \mu_t \cdot m_b(t),$$

where $\mu_t = \max_{s \in [t-\tau, t]} \hat{\mu}(s)$ and $\hat{\mu}(s) = \mu(s)$ for $s \ge 0$, $\hat{\mu}(s) = \mu(0)$ for $s < 0$.

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