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ON A CLASS OF ADDITIVE GROUP ACTIONS ON AFFINE THREE-SPACE

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ABSTRACT. Every algebraic action of the additive group of complex numbers on complex affine space is obtained as the exponential of a locally nilpotent derivation on its coordinate ring. Moreover, a locally nilpotent derivation is equivalent to a polynomial vector field on affine space admitting a strictly polynomial flow. For three-dimensional affine space it is known that the group action is triangulable if and only if the centralizer of the corresponding vector field, in the Lie algebra of vector fields on affine space, contains a constant vector field. A class of additive group actions is investigated with this criterion, and the generic member is shown to be nontriangulable.

1. Introduction. In [**13**] Rentschler showed that, for any field K of characteristic zero, all algebraic actions of the additive group of K on the affine plane over K are triangulable. The corresponding assertion in higher dimensions is false, as first demonstrated by Bass in [**1**]. Since this example first appeared, several authors have found other nontriangulable actions of the additive group of complex numbers, henceforth denoted by G_a, or complex affine three space [**12, 3, 2, 9, 10**]. The interest in these examples stems from the attempts to understand the structure of the automorphism group of complex affine space, as an infinite dimensional algebraic group, by investigating the homomorphisms to it from well understood, finite dimensional, algebraic groups. Indeed, the Jung–van der Kulk theorem [**11**] can be viewed as accomplishing this for the complex plane, and the theorem of Rentschler can be viewed as a nonlinear Lie-Kolchin theorem. However, the nontriangulable G_a actions on complex three space indicate that no Lie-Kolchin theorem holds in dimension higher than two, and that the structure of the automorphism group is far more complicated in these dimensions.

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