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SOME CHARACTERIZATIONS FOR BOX SPLINE WAVELETS AND LINEAR DIOPHANTINE EQUATIONS

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ABSTRACT. Box splines are investigated from the point of view of wavelets. Some characterizations concerning linear independence of integer translates of Box splines are presented in terms of the defining matrices. It is shown that a direct extension of a criterion for linear independence of refinable functions in the univariate case to the multivariate case holds for the Box spline M_Ξ in $\mathbf{R}^s$ when $\operatorname{rank}\Xi = s$ while not any more when $\operatorname{rank}\Xi < s$.

1. Introduction and main results. Stability and linear independence of integer translates of a refinable function or distribution play basic roles in wavelet decompositions and multivariate splines. These properties can be characterized by the Fourier-Laplace transform of this distribution. It was shown by Ron [**17**] that, for a compactly supported distribution ϕ in $\mathbf{R}^s$, $\{\phi(\cdot - \alpha) : \alpha \in \mathbf{Z}^s\}$ are linearly independent if and only if, for any $\omega \in \mathbf{C}^s$, there exists some $\alpha \in \mathbf{Z}^s$ such that $\phi^\wedge(\omega + 2\pi\alpha) \neq 0$, where $\phi^\wedge$ is the Fourier-Laplace transform of ϕ.

Suppose that ϕ is k-refinable, $2 \leq k \in \mathbf{N}$, say

$$\phi = \sum_{\alpha \in \mathbf{Z}^s} b_\alpha \phi(k \cdot - \alpha), \tag{1.1}$$

$$\phi^\wedge(0) = 1, \tag{1.2}$$

where $\{b_\alpha\}_{\alpha \in \mathbf{Z}^s}$ is a finitely supported sequence called the mask sequence of the refinement equation (1.1). Then ϕ can be determined by

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