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# RELATIONSHIPS AMONG THE FIRST VARIATION, THE CONVOLUTION PRODUCT, AND THE FOURIER-FEYNMAN TRANSFORM

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ABSTRACT. In this paper we examine the various relationships that exist among the first variation, the Fourier-Feynman transform, and the convolution product for functionals on Wiener space that belong to a Banach algebra $\mathcal{S}$.

**1. Introduction.** Let $C_0[0,T]$ denote one-parameter Wiener space; that is the space of $\mathbf{R}$-valued continuous functions on $[0,T]$ with $x(0)=0$. The concept of an $L_1$ analytic Fourier-Feynman transform was introduced by Brue in [**1**]. In [**3**], Cameron and Storvick introduced an $L_2$ analytic Fourier-Feynman transform. In [**12**], Johnson and Skoug developed an $L_p$ analytic Fourier-Feynman transform theory for $1 \leq p \leq 2$ which extended the results in [**1, 3**] and gave various relationships between the $L_1$ and $L_2$ theories. In [**9**], Huffman, Park and Skoug defined a convolution product for functionals on Wiener space and in [**9, 10, 11**] obtained various results involving the Fourier-Feynman transform and the convolution product.

The class of functionals on $C_0[0,T]$ that we work with throughout this paper is the Banach algebra $\mathcal{S}$ introduced by Cameron and Storvick in [**4**]. Results in [**7, 8, 14, 15**] show that $\mathcal{S}$ contains many broad subclasses of functionals of interest in connection with Feynman integration theory and quantum mechanics.

In Section 3 of this paper we examine all relationships involving exactly two of the three concepts of "transform," "convolution product" and "first variation" of functionals in $\mathcal{S}$. In Section 4, we examine all relationships involving all three of these concepts where each concept is used exactly once. Study of these relationships yields many interesting formulas; see, for example, equations (3.7), (3.9), (4.1), (4.3) and (4.7).

**2. Definitions and preliminaries.** Let $\mathcal{M}$ denote the class of all Wiener measurable subsets of $C_0[0,T]$, and let $m$ denote Wiener

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