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PLANCHEREL THEOREM FOR VECTOR VALUED FUNCTIONS AND BOEHMIANS

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ABSTRACT. The classical Plancherel theorem asserts that the Fourier-Plancherel transform is an isomorphism between $\mathcal{L}^2(\mathbf{R})$ onto $\mathcal{L}^2(\mathbf{R})$. On the other hand, in the literature the theory of Fourier transform is extended to the space of $\mathcal{L}^1$ Boehmians and also to the space of tempered Boehmians. In this paper we shall introduce two types of Boehmians, each of which contains vector valued square integrable functions on $\mathbf{R}$ as a dense subspace and extend the theory of Fourier transform to this set up. Finally we prove that this extended Fourier transform is a one-to-one continuous linear map of one space of Boehmians onto the other.

1. Introduction. The theory of Schwartz distributions, tempered distributions and their applications are well known in the literature. The concept of Boehmians which was motivated by Boehme's regular operators [**1**] was defined and systematically developed and their properties investigated in [**2, 4, 5, 7, 12, 13**]. Further several integral transforms were also introduced on various spaces of Boehmians and their properties studied in [**5, 6, 8, 9, 10, 14**].

In [**19**], Zemanian develops the theory of Laplace transform for a testing function space consisting of Banach space valued functions defined on $\mathbf{R}^k$. Motivated by the above theory, in this paper we shall develop a theory of Fourier transform on a certain space of Boehmians which contains vector valued functions defined on $\mathbf{R}^1$. Let us first consider a separable commutative Banach algebra A with identity e. It may still be possible to take just a separable Banach space instead of a Banach algebra but that may lead to more complications, and we shall return to this problem later. To develop our theory we need an analogue of the Plancherel theorem on the space $\mathcal{L}^2(A)$ where

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