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SIMULTANEOUS APPROXIMATION BY BIRKHOFF INTERPOLATORS

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Dedicated to the late Arun Kumar VARMA

1. Introduction and problem description. In the present paper we give general inequalities for simultaneous approximation by Birkhoff interpolators (provided the underlying problem is regular).

Let

$$\Delta_n : -1 \leq x_n < x_{n-1} < \cdots < x_1 \leq 1$$

be a sequence of arbitrary points. With this sequence of points we associate an incidence matrix

$$E = (e_{i,j})_{i=1,\dots,n;j=0,\dots,R}$$

where R is a positive integer. Such matrices have as entries $|E| \geq n$ ones and $n(R+1) - |E|$ zeros and are such that in each row there is at least one entry equal to one. We also assume that the last column contains at least one entry equal to one. The Birkhoff interpolation problem consists of finding a polynomial P of degree $|E|-1$ such that the following $|E|$ interpolation conditions are fulfilled:

$$P^{(j)}(x_i) = a_i^{(j)} \quad \text{if } e_{i,j} = 1.$$

Here the $a_i^{(j)}$ are arbitrary real numbers.

The pair (E, Δ_n) is called *regular* if, for each choice of the $a_i^{(j)}$, such a polynomial exists and is uniquely determined. In this case there exist uniquely determined fundamental functions $A_{i,j} \in \prod_{|E|-1}$ such that the interpolating polynomial can be written as

$$P(x) = \sum_{e_{i,j}=1} a_i^{(j)} \cdot A_{i,j}(x).$$

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