ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 28, Number 4, Winter 1998

POLYNOMIAL COMPACTNESS IN BANACH SPACES

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ABSTRACT. We investigate infinite dimensional Banach spaces equipped with the initial topology with respect to the continuous polynomials. We show nonlinear properties for this topology in both the real and the complex case. A new property for Banach spaces, polynomial Dunford-Pettis property, is introduced. For spaces with this property the compact sets in the topology induced by the polynomials are shown to be invariant under the summation map. For most real Banach spaces we characterize the polynomially compact sets as the bounded sets that are separated from zero by the positive polynomials.

Denote a Banach space X equipped with the topology induced by its continuous polynomials by $X_{\mathcal{P}(X)}$. This article investigates the topological space $X_{\mathcal{P}(X)}$ with a focus on its compact sets. In [**3**] Aron et al. prove that $X_{\mathcal{P}(X)}$ has a nonlinear topology if X is an infinite dimensional complex Hilbert space. We show that there are also real as well as entirely other complex Banach spaces, e.g., ℓ^∞, with nonlinear polynomial topologies. Although $X_{\mathcal{P}(X)}$ is not linear in general, we show that the compact sets in $X_{\mathcal{P}(X)}$ form an invariant class under the sum operation for large classes of spaces X. This is shown to be the case when X has the property (P) studied in [**2**] by Aron et al., or when X is a $\mathcal{P}$-Dunford-Pettis space, a new class of spaces containing all the Dunford-Pettis spaces and all the Λ-spaces. We investigate this class of $\mathcal{P}$-Dunford-Pettis spaces with emphasis on its connections with the polynomial Dunford-Pettis properties studied by Farmer and Johnson [**16**] as well as the Dunford-Pettis-like properties of Castillo and Sánchez [**9**].

For real Banach spaces X, we give an almost covering characterization of the relatively compact sets in $X_{\mathcal{P}(X)}$ as those bounded sets that are separated from zero by all strictly positive polynomials in $\mathcal{P}(X)$.

Received by the editors on September 1, 1995.
1992 AMS *Mathematics Subject Classification.* 46E25, 46J10.
Key words and phrases. Polynomials, Dunford-Pettis property, compactness, homomorphisms, nonlinear topology.
Research partially supported by DGICYT grant PB-0044 (Spain).